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HW 4.2 Changing the Period of Sine and Cosine Functions

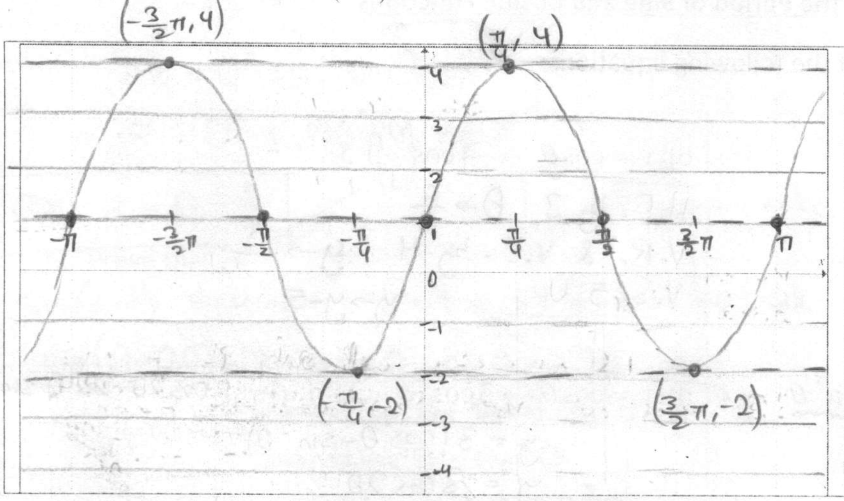
1. Indicate the transformations for each of the following equations:

a) $y=\cos\theta \to y=-4\cos 3\theta$ H.C. by $\frac{1}{3}$ $\theta\to 3\theta$ V.R. and V.E. by 4 $y\to -\frac{1}{4}y$	b) $y=\cos\theta \to -4\cos\left(\frac{\theta}{3}\right)+5$ H.E. by 3 $\theta \to \frac{\theta}{3}$ V.R. & V.E. by 4 $y\to -\frac{1}{4}y$ V.S. 5 Up $y\to y-5$
c) $y=\sin\theta \to y=\cos^2\theta-3\sin\theta\cos\theta+\sin^2\theta$ $\sin^2\theta+\cos^2\theta=1$ $\sin 2\theta = 2\sin\theta\cos\theta \Rightarrow -\frac{3}{2}\sin 2\theta = -3\sin\theta\cos\theta$ $y=\sin\theta \to y=-\frac{3}{2}\sin 2\theta+1$ H.C. by $\frac{1}{2}$ $\theta\to 2\theta$ V.R. and V.E. by $\frac{3}{2}$ $y\to -\frac{2}{3}y$ V.S. 1U $y\to y-1$	d) $y=\cos\theta \to 3\cos^2\theta-3\sin^2\theta$ $\cos 2\theta=\cos^2\theta-\sin^2\theta$ $y=3(\cos^2\theta-\sin^2\theta)$ $y=3\cos 2\theta$ H.C. by $\frac{1}{2}$ $\theta\to 2\theta$ V.E. by 3 $y\to \frac{1}{3}y$
e) $y=\cos\theta \to y=\cos^2\theta+2$ $\cos 2\theta = \cos^2\theta-\sin^2\theta = \cos^2\theta-1+\cos^2\theta = 2\cos^2\theta-1$ $\sin^2\theta = 1-\cos^2\theta$ $\frac{\cos 2\theta+5}{2} = \cos^2\theta+2$ H.C. by $\frac{1}{2}$, V.S. 5U, V.C. by $\frac{1}{2}$	f) $y=\sin\theta \to y=4\sin^2\theta-3$ $\sin\theta=\cos\left(\frac{\pi}{2}+\theta\right)$ $\cos\left(\frac{\pi}{2}+\theta\right)\to\cos(\theta)$ $\cos(\theta)\to\cos(2\theta)$ $=2\cos^2\theta-1$ $2\cos^2\theta-1\to 4\cos^2\theta-2$ $4\cos^2\theta-2\to 4\cos^2\theta-3$ H.S. $\frac{\pi}{2}$ R H.C. by $\frac{1}{2}$ V.E. by 2 V.S. 1L

2. Indicate the period for each of the following functions:

a) $y=3\cos\frac{3\theta}{2}+1$ $y=A\cos(B\theta+C)+1$ Period: $\frac{2\pi}{B}$ $\leftarrow B=\frac{3}{2}$ Period: $\frac{2\pi}{\frac{3}{2}} = \boxed{\frac{4\pi}{3}}$	b) $y=2\sin 3\theta-4$ Period: $\boxed{\frac{2\pi}{3}}$
c) $y=3\sin 2\theta\cos 2\theta = \frac{3}{2}\sin 4\theta$ $\sin 2\theta = 2\sin\theta\cos\theta$ Period: $\frac{2\pi}{4} = \boxed{\frac{1}{2}\pi}$	d) $y=\cos^2 3\theta-\sin^2 3\theta = \cos 6\theta$ $\cos^2\theta-\sin^2\theta=\cos 2\theta$ Period: $\frac{2\pi}{6} = \boxed{\frac{\pi}{3}}$
e) $y=\sqrt{\frac{1-\cos\theta}{2}}$ $\cos 2\theta = 1-2\sin^2\theta$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$ Period: $\frac{2\pi}{\frac{1}{2}} = \boxed{4\pi}$	f) $y=\frac{\sin\theta}{1+\cos\theta}$ $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$ Period: $\frac{\pi}{\frac{1}{2}} = \boxed{2\pi}$

3. Graph the function on the graph provided. Set and label the increments to have at least two cycle of the function: $y = 3\sin(2\theta) + 1$

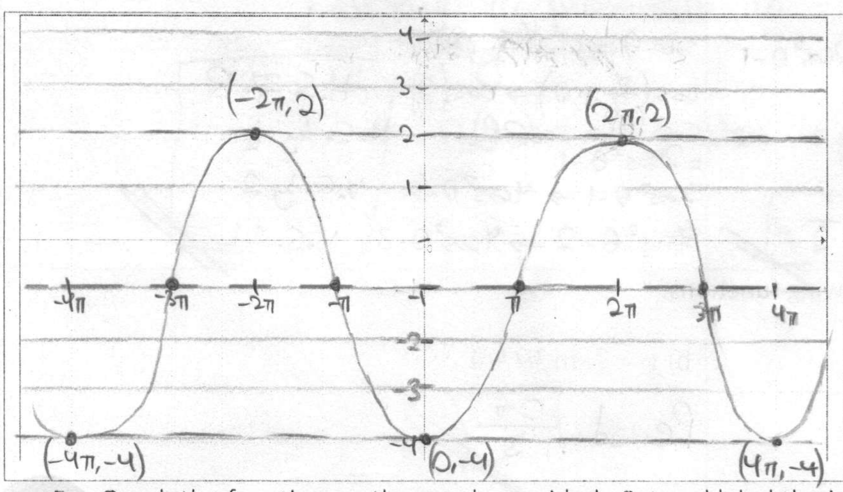


i) Label the maximum and minimum points on the graph (provide coordinates)

ii) What is the period of the function?

Period $= \frac{2\pi}{2} = \boxed{\pi}$

4. Graph the function on the graph provided. Set and label the increments to have at least two cycle of the function: $y = -3\cos\left(\frac{\theta}{2}\right) - 1$

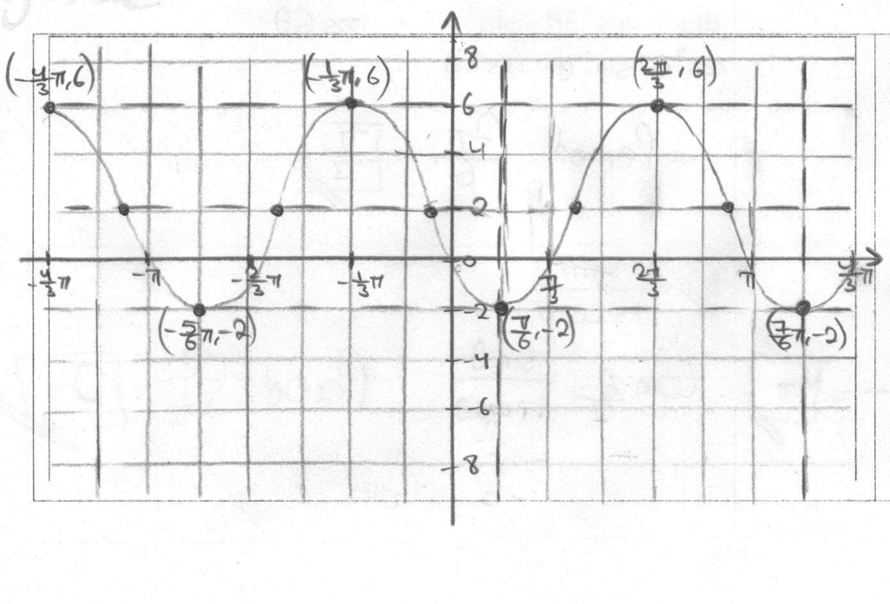


i) Label the maximum and minimum points on the graph (provide coordinates)

ii) What is the period of the function?

Period $= \frac{2\pi}{\frac{1}{2}} = \boxed{4\pi}$

5. Graph the function on the graph provided. Set and label the increments to have at least two cycle of the function: $y = -4\cos\left(2\theta - \frac{\pi}{3}\right) + 2$ $\Rightarrow y = -4\cos\left(2\left(\theta - \frac{\pi}{6}\right)\right)$

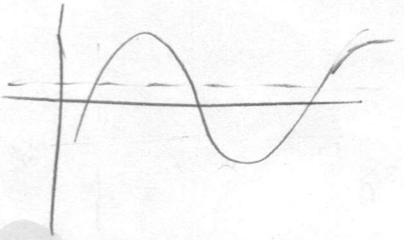


i) Label the maximum and minimum points on the graph (provide coordinates)

ii) What is the period of the function?

Period $= \frac{2\pi}{2} = \boxed{\pi}$

6. Given the equation $0.5 = a\cos b\theta + d$, how many solutions will there be for $0° \leq \theta \leq 360°$ in terms of "a", "b", and "d"

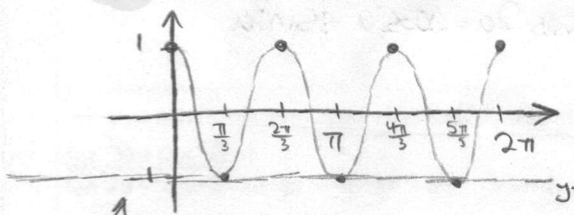


if $0.5 < d - |a|$ or $0.5 > d + |a|$ then 0 sols

if $0.5 = d - |a|$ or $0.5 = d + |a|$ then "b" number of sols.

if $0.5 > d - |a|$ or $0.5 < d + |a|$ then "2b" solutions

7. Given the equation $\sin 3\theta = k$, for what values of "k" will there be only three solutions $0° \leq \theta < 360°$



Period: $\frac{2}{3}\pi$

With $0° \leq \theta < 360°$, there will only be 3 solutions at the max and min. Therefore, $\boxed{k = 1, -1}$

8. Given the equation $\sin 3\theta = k$, for what range in values for "k" will there be only six solutions $0° \leq \theta < 360°$

Look at graph above, if $\boxed{-1 < k < 1}$, then there will only be 6 solutions for $0° \leq \theta < 360°$.

9. Which two of the functions below are the same function?

$i) y = 3\sin 2(\theta + \frac{\pi}{2})$ $ii) y = 3\cos 2\theta$ $iii) y = 3\cos 2(\theta + \frac{\pi}{4})$ $iv) y = 3\sin 2(\theta + \pi)$

$y = 3\sin(2\theta + \pi)$

$y = 3\cos(2\theta + \frac{\pi}{2})$

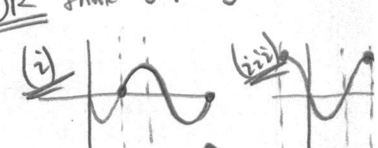
$y = 3\sin(2\theta + \pi)$

$y = 3\sin(2\theta + 2\pi)$

$\sin(\frac{\pi}{2} + \theta) = \cos\theta$

(i) and (iii) are the same function.

OR think graphically



10. Which two of the functions below are the same function?

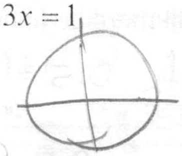
$i) y = 3\sin 2(2\theta + \pi)$ $ii) y = 3\cos(2\theta + \frac{\pi}{2})$ $iii) y = 3\cos 2\theta$ $iv) y = 3\sin 2(2\theta + 2\pi)$

$= 3\sin(4\theta + 2\pi)$

$y = 3\sin(4\theta + 4\pi)$

(i) and (iv) are the same function as they have the same period but different phase shifts that are multiples of the period.

11. Evaluate $\cos x$ if $\cos 3x = 1$



$\cos 3x = 1$

$3x = 0^\circ$

$x = 0^\circ \pm 120^\circ n \; ; n \in \mathbb{Z}$

$\boxed{\cos x = 0, -\frac{1}{2}}$

supposed to be $-4\sin^{3}\theta$ not 2

12. Prove that $\sin 3\theta = 3\sin\theta - 4\sin^2\theta$

$\sin(a+b) = \sin a\cos b + \sin b\cos a \qquad \sin 2a = 2\sin a\cos a \qquad \cos 2a = \cos^2 a - \sin^2 a$

$\sin(\theta + 2\theta) = \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta(1-\sin^2\theta) - \sin^3\theta$

$\boxed{= 3\sin\theta - 4\sin^3\theta}$

13. Use the identity above to prove that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Solution provided in Math 10/11 Honours Trig HW.

(This exact problem appeared there too)